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NEUTRINO FLAVOR DETECTION AT NEUTRINO TELESCOPES AND ITS USES

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1. Introduction

We assume that sources of high energy astrophysical neutrinos exist with detectable fluxes. The existence of gamma ray sources with energies of upto 100s of TeV provides some evidence that this assumption may be correct¹, assuming that the gamma rays are coming from π^0 's. We also need to assume that in the not too distant future, large enough detectors, well instrumented and with good angular and energy resolution will be operating(hopefully of several KM3 size)^{2,3,4}. We assume also that neutrino signals will be seen, and furthermore that the detectors will have the ability to distinguish flavors(at the moment this is only assured for H_2O detectors). If all these optimistic assumptions turn out to be valid, there are a number of uses that these detectors can be put to⁵. I would like to discuss some of them.

2. Astrophysical neutrino flavor content

Since most of the sources are tenous and emit neutrinos from π/K decays via the $\pi - \mu - e$ chain, we expect that the ratio of ν_e to ν_μ is 1:2; furthermore, estimates of ν_τ production even at very high energies yield very small fraction of ν_τ 's⁶. Hence, for practical purposes the flavor ratio produced in such "conventional" sources(either via p-p or γ -p collisions) is $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$. The mixture of ν 's and $\bar{\nu}$'s is expected to be 1:1 with some exceptions. There are also sources in which muons lose energy by magnetic fields or other means(called damped muon sources); in this case the flavor mix becomes $\nu_e : \nu_\mu : \nu_\tau = 0 : 1 : 0$, this can be an energy dependent effect, making the flavor mix energy dependent⁷. There may be sources, in which the dominant component is from neutron decays, resembling a "beta beam"⁸, with the resultant mix being: $\nu_e : \nu_\mu : \nu_\tau = 1 : 0 : 0$. If the density is high enough for pions to interact before decaying, then heavy flavors dominate and the flavor mix becomes: $\nu_e : \nu_\mu : \nu_\tau = 1 : 1 : 0$. This is the case,for example in the so-called slow-jet supernovas⁹.

There are also the neutrinos emitted as a by product in the GZK¹⁰ reaction, properly called the BZ(Berezinsky-Zatsepin)¹¹ neutrinos. The reaction is of course

$p + \gamma \rightarrow \Delta^+ \rightarrow n + \pi^+$. In this case the flavor mix depends on the energy¹². Below about 100 PeV, it is a pure "beta beam" with $\nu_e : \nu_\mu : \nu_\tau = 1 : 0 : 0$; and above 100 PeV it is conventional $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$.

3. Effect of Oscillations

The current knowledge of neutrino masses and mixing¹³ can be summarized as follows¹⁴. The mixing matrix elements are given to a very good approximation by the so-called tri-bi-maximal matrix¹⁵. The bound on the element $|U_{e3}|$ comes from the CHOOZ experiment¹⁶ and is given by $|U_{e3}| < 0.17$. The mass spectrum has two possibilities: normal or inverted. The mass differences are given by $|\delta m_{32}^2| \sim 2.4 \cdot 10^{-3} eV^2$ (with the + sign corresponding to normal hierarchy and - sign to the inverted one) and $\delta m_{21}^2 \sim +7.6 \cdot 10^{-5} eV^2$. Since $\delta m^2 L / 4E$ for the distances to GRB's and AGN's (even for energies up to and beyond PeV) is very large ($> 10^7$) the oscillations have always averaged out and the conversion(or survival) probability is given by

$$P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2 \quad (1)$$

Assuming no significant matter effects en-route, it is easy to show that the tri-bi-maximal mixing matrix leads to a simple propagation matrix P , which, for any value of the solar mixing angle, converts a flux ratio of $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$ into one of $1 : 1 : 1$. Hence the flavor mix expected at arrival is simply an equal mixture of ν_e, ν_μ and ν_τ as was observed long ago^{6,17}. Similarly, for the other cases neutrino mixing modifies the flavor mixes as:

	Initial	After Mixing
Damped Muon	(0:1:0)	(4:7:7)
Beta Beam	(1:0:0)	(5:2:2)
Prompt	(1:1:0)	(14:11:11)

If the universal flavor mix is confirmed by future observations, our current knowledge of neutrino masses and mixing is reinforced and conventional wisdom about the beam dump nature of the production process is confirmed as well. However, it would be much more exciting to find deviations from it, and learn something new. How can this come about? I give below a shopping list of variety of ways in which this could come to pass, and what can be learned in each case.

4. Discriminating Flavors

We define two ratios to distinguish various flavor mixes as: $f_e (= e/(e + \mu + \tau))$ and $R (= \mu/(e + \tau))$. Then we have the following for the various possible flavor mixes expected at earth from various source types:

Source Type	f_e	R
Pionic	0.33	0.5
Damped-muon	0.22	0.64
Beta-Beam	0.55	0.29
Prompt	0.39	0.44

It has been shown that R and/or f_e can be determined up to an accuracy of about 0.08 or so in an ice-cube type detector¹⁸. Hence, pionic, damped muon and Beta-beam type os sources can be distinguished but probably not the prompt.

There have been many suggestions that small deviations from the canonical flavor mixes can be used for a variety of purposes, such as determine small deviations from Tri-bi-maximal mixing(i.e. measure θ_{13} and δ), small mixing with sterile neutrinos etc¹⁹. However, there are several reasons why this is rather impractical. In addition to the limits on the precision with which f_e and/or R can be measured, there are inherent uncertainties in the source flavor mixes themselves. For example, in the π/K case the flavor mix is not expected to be exactly $\nu_e : \nu_\mu : \nu_\tau = 1 : 2 : 0$ but rather more like 1:1.85:0²⁰. The main reason for this effect is the muon polarization in the $\pi - \mu$ decay which gives rise to a ν_μ of lower energy than ν_e , with additional subtle effects in K-decays. Similarly, the flavor mixes in the damped muon case and Beta-beam case are also not expected to give rise to the simple pure flavor mixes. These effects combined with the 8 % uncertainty in the experimental determination of the flavor mix renders extremely difficult any attempt to measure small effects in mixing²¹.

5. Deviation from Canonical Mix

Let us consider how the conventional mix may undergo major deviations which are detectable. The possibility that the mass differences between neutrino mass eigenstates are zero in vacuum (and become non-zero only in the presence of matter) has been raised²². If this is true, then the final flavor mix should be the same as initial, namely: 1 : 2 : 0. However, very recently, analysis of low energy atmospheric neutrino data by Super-Kamiokande has ruled out a wide variety of models for such behavior²³.

Neutrino decay is another important possible way for the flavor mix to deviate significantly from the democratic mix²⁴. We now know that neutrinos have non-zero masses and non-trivial mixing, based on the evidence for neutrino mixing and oscillations from the data on atmospheric, solar and reactor neutrinos.

Once neutrinos have masses and mixing, then in general, the heavier neutrinos are expected to decay into the lighter ones via flavor changing processes²⁵. The only remaining questions are (a) whether the lifetimes are short enough to be phenomenologically interesting (or are they too long?) and (b) what are the dominant decay modes. Since we are interested in decay modes which are likely to have rates (or lead to lifetimes) which are phenomenologically interesting, we can rule out several classes of decay modes immediately. For example, the very strong constraints²⁵

on radiative decay modes and on three body modes such as $\nu \rightarrow 3\nu$ render them uninteresting.

The only decay modes which can have interestingly fast decay rates are two body modes such as $\nu_i \rightarrow \nu_j + x$ where x is a very light or massless particle, e.g. a Majoron. In general, the Majoron is a mixture of the Gelmini- Roncadelli²⁶ and Chikasige-Mohapatra- Peccei²⁷ type Majorons. Explicit models of this kind which can give rise to fast neutrino decays have been discussed²⁸. The models with $\Delta L = 2$ are unconstrained by μ and τ decays which cannot be engendered by such couplings. Both($\Delta L = 2$ and $\Delta L = 0$) kinds of models with couplings of ν_μ and ν_e are constrained by the limits on multi-body π , K decays, and on $\mu - e$ universality violation in π and K decays²⁹, but these bounds allow fast neutrino decays.

Direct limits on such decay modes are rather weak. Current bounds on such decay modes are as follows. For the mass eigenstate ν_1 , the limit is about

$$\tau_1 \geq 10^5 \text{ sec/eV} \quad (2)$$

based on observation of $\bar{\nu}_e$ s from SN1987A³⁰ (assuming CPT invariance). For ν_2 , strong limits can be deduced from the non-observation of solar anti-neutrinos in KamLAND³¹. A more general but similar bound is obtained from an analysis of solar neutrino data³². This bound is given by:

$$\tau_2 \geq 10^{-4} \text{ sec/eV} \quad (3)$$

For ν_3 , one can derive a bound from the atmospheric neutrino observations of upcoming neutrinos³³:

$$\tau_3 \geq 10^{-10} \text{ sec/eV} \quad (4)$$

The strongest lifetime limit is thus too weak to eliminate the possibility of astrophysical neutrino decay by a factor about $10^7 \times (L/100 \text{ Mpc}) \times (10 \text{ TeV}/E)$. It was noted that the disappearance of all states except ν_1 would prepare a beam that could in principle be used to measure elements of the neutrino mixing matrix³⁴, namely the ratios $|U_{e1}|^2 : |U_{\mu 1}|^2 : |U_{\tau 1}|^2$. The possibility of measuring neutrino lifetimes over long baselines was mentioned in Ref.³⁵, and some predictions for decay in four-neutrino models were given in Ref.³⁷. The particular values and small uncertainties on the neutrino mixing parameters allow for the first time very distinctive signatures of the effects of neutrino decay on the detected flavor ratios. The expected increase in neutrino lifetime sensitivity (and corresponding anomalous neutrino couplings) by several orders of magnitude makes for a very interesting test of physics beyond the Standard Model; a discovery would mean physics much more exotic than neutrino mass and mixing alone. Because of its unique signature, neutrino decay cannot be mimicked by either different neutrino flavor ratios at the source or other non-standard neutrino interactions.

A characteristic feature of decay is its strong energy dependence: $\exp(-Lm/E\tau)$, where τ is the rest-frame lifetime. For simplicity, we will consider the case that decays are always complete, i.e., that these exponential factors vanish. The simplest

case (and the most generic expectation) is a normal hierarchy in which both ν_3 and ν_2 decay, leaving only the lightest stable eigenstate ν_1 . In this case the flavor ratio is³⁴ $|U_{e1}|^2 : |U_{\mu 1}|^2 : |U_{\tau 1}|^2$. Thus, if $|U_{e3}| = 0$ we have

$$\phi_{\nu e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} \simeq 4 : 1 : 1, \quad (5)$$

where we used the propagation matrix derived from the tri-bi-maximal mixing. Note that this is an extreme deviation of the flavor ratio from that in the absence of decays. It is difficult to imagine other mechanisms that would lead to such a high ratio of ν_e to ν_μ . In the case of inverted hierarchy, ν_3 is the lightest and hence stable state, and so²⁴ we have instead

$$\phi_{\nu e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} = |U_{e3}|^2 : |U_{\mu 3}|^2 : |U_{\tau 3}|^2 = 0 : 1 : 1. \quad (6)$$

If $|U_{e3}| = 0$ and $\theta_{atm} = 45^0$, each mass eigenstate has equal ν_μ and ν_τ components. Therefore, decay cannot break the equality between the ϕ_{ν_μ} and ϕ_{ν_τ} fluxes and thus the $\phi_{\nu e} : \phi_{\nu_\mu}$ ratio contains all the useful information.

When $|U_{e3}|$ is not zero, and the hierarchy is normal, it is possible to obtain information on the values of $|U_{e3}|$ as well as the CPV phase δ ³⁶. The flavor ratio e/μ varies from 4 to 10 (as $|U_{e3}|$ goes from 0 to 0.2) for $\cos \delta = +1$ but from 4 to 2.5 for $\cos \delta = -1$. The ratio τ/μ varies from 1 to 4 ($\cos \delta = +1$) or 1 to 0.25 ($\cos \delta = -1$) for the same range of U_{e3} .

If the decays are not complete and if the daughter does not carry the full energy of the parent neutrino; the resulting flavor mix is somewhat different but in any case it is still quite distinct from the simple 1 : 1 : 1 mix²⁴. There is a very recent exhaustive study of the various possibilities³⁸.

If the path of neutrinos takes them thru regions with significant magnetic fields and the neutrino magnetic moments are large enough, the flavor mix can be affected³⁹. The main effect of the passage thru magnetic field is the conversion of a given helicity into an equal mixture of both helicity states. This is also true in passage thru random magnetic fields⁴⁰. It has been shown recently that the presence of a magnetic field of a few(10 or more) Gauss at the source can make the neutrinos decohere as they traverse cosmic distances⁴¹.

If the neutrinos are Dirac particles, and all magnetic moments are comparable, then the effect of the spin-flip is to simply reduce the overall flux of all flavors by half, the other half becoming the sterile Dirac partners. If the neutrinos are Majorana particles, the flavor composition remains 1 : 1 : 1 when it starts from 1 : 1 : 1, and the absolute flux remains unchanged.

Other neutrino properties can also affect the neutrino flavor mix and modify it from the canonical 1 : 1 : 1. If neutrinos have flavor(and equivalence principle) violating couplings to gravity(FVG); then there can be resonance effects which make for one way transitions(analogues of MSW transitions) e.g. $\nu_\mu \rightarrow \nu_\tau$ but not vice versa^{42,43}. In case of FVG for example, this can give rise to an anisotropic deviation of the ν_μ/ν_τ ratio from 1, becoming less than 1 for events coming from the direction towards the Great Attractor, while remaining 1 in other directions⁴².

If such striking effects are not seen, then the current bounds on such violations can be improved by six to seven orders of magnitude.

Complete quantum decoherence would give rise to a flavor mix given by 1 : 1 : 1, which is identical to the case of averaged out oscillations as we saw above. The distinction is that complete decoherence always leads to this result; whereas averaged out oscillations lead to this result only in the special case of the initial flavor mix being 1 : 2 : 0. To find evidence for decoherence, therefore, requires a source which has a different flavor mix. One possible practical example is the “beta” beam source with an initial flavor mix of 1 : 0 : 0. In this case decoherence leads to the universal 1 : 1 : 1 mix whereas the averaged out oscillations lead to 2.5 : 1 : 1⁴⁴. The two cases can be easily distinguished from each other.

Violations of Lorentz invariance and/or CPT invariance can change the final flavor mix from the canonical universal mix of 1 : 1 : 1 significantly. With a specific choice of the change in dispersion relation due to Lorentz Invariance Violation, the effects can be dramatic. For example, the final flavor mix at sufficiently high energies can become 7 : 2 : 0⁴⁴.

If each of the three neutrino mass eigenstates is actually a doublet with very small mass difference (smaller than $10^{-6} eV$), then there are no current experiments that could have detected this. Such a possibility was raised long ago⁴⁵. It turns out that the only way to detect such small mass differences ($10^{-12} eV^2 > \delta m^2 > 10^{-18} eV^2$) is by measuring flavor mixes of the high energy neutrinos from cosmic sources. Relic supernova neutrino signals and AGN neutrinos are sensitive to mass difference squared down to $10^{-20} eV^2$ ⁴⁶.

Let $(\nu_1^+, \nu_2^+, \nu_3^+; \nu_1^-, \nu_2^-, \nu_3^-)$ denote the six mass eigenstates where ν^+ and ν^- are a nearly degenerate pair. A 6x6 mixing matrix rotates the mass basis into the flavor basis. For pseudo-Dirac neutrinos, Kobayashi and Lim⁴⁷ have given an elegant proof that the 6x6 matrix V_{KL} takes the very simple form (to lowest order in $\delta m^2/m^2$):

$$V_{KL} = \begin{pmatrix} U & 0 \\ 0 & U_R \end{pmatrix} \cdot \begin{pmatrix} V_1 & iV_1 \\ V_2 & -iV_2 \end{pmatrix}, \quad (7)$$

where the 3×3 matrix U is just the usual mixing (MNSP)matrix determined by the atmospheric and solar observations, the 3×3 matrix U_R is an unknown unitary matrix and V_1 and V_2 are the diagonal matrices $V_1 = \text{diag}(1, 1, 1)/\sqrt{2}$, and $V_2 = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, e^{-i\phi_3})/\sqrt{2}$, with the ϕ_i being arbitrary phases.

The flavor ratios deviate from 1 : 1 : 1 when one or two of the pseudo-Dirac oscillation modes is accessible. In the ultimate limit where L/E is so large that all three oscillating factors have averaged to $\frac{1}{2}$, the flavor ratios return to 1 : 1 : 1, with only a net suppression of the measurable flux, by a factor of 1/2. As a bonus, if such small pseudo-Dirac mass differences exist, it would enable us to measure cosmological parameters such as the red shift in neutrinos(rather than in photons)^{35,46}.

6. Experimental Flavor Identification

It is obvious from the above discussion that flavor identification is crucial for the purpose at hand. In a water(or ice) cerenkov detector flavors can be identified as follows.

The ν_μ flux can be measured by the μ' s produced by the charged current interactions and the resulting μ tracks in the detector which are long at these energies. ν'_e s produce showers by both CC and NC interactions. The total rate for showers includes those produced by NC interactions of ν'_μ s and ν'_τ s as well and those have to be (and can be) subtracted off to get the real flux of ν'_e s. Double-bang and lollipop events are signatures unique to ν'_τ s, made possible by the fact that tau leptons decay before they lose a significant fraction of their energy. A double-bang event consists of a hadronic shower initiated by a charged-current interaction of the ν_τ followed by a second energetic shower from the decay of the resulting τ^6 . A lollipop event consists of the second of the double-bang showers along with the reconstructed tau lepton track (the first bang may be detected or not). In principle, with a sufficient number of events, a fairly good estimate of the flavor ratio $\nu_e : \nu_\mu : \nu_\tau$ can be reconstructed (with caveats about uncertainties) as has been discussed recently. Deviations of the flavor ratios from 1 : 1 : 1 due to possible decays are so extreme that they should be readily identifiable¹⁸. High energy neutrino telescopes, such as Icecube⁴⁸, will not have perfect ability to separately measure the neutrino flux in each flavor. However, the situation is salvageable. In the limit of $\nu_\mu - \nu_\tau$ symmetry the fluxes for ν_μ and ν_τ are always in the ratio 1 : 1, with or without decay. This is useful since the ν_τ flux is the hardest to measure.

Even when the tau events are not all identifiable, the relative number of shower events to track events can be related to the most interesting quantity for testing decay scenarios, i.e., the ν_e to ν_μ ratio. The precision of the upcoming experiments should be good enough to test the extreme flavor ratios produced by decays. If electromagnetic and hadronic showers can be separated, then the precision will be even better¹⁸. Comparing, for example, the standard flavor ratios of 1 : 1 : 1 to the possible 4 : 1 : 1 (or 0 : 1 : 1 for inverted hierarchy)generated by decay, the higher(lower) electron neutrino flux will result in a substantial increase(decrease) in the relative number of shower events. The measurement will be limited only by the energy resolution of the detector and the ability to reduce the atmospheric neutrino background(which drops rapidly with energy and should be negligibly small at and above the PeV scale).

7. Discussion and Conclusions

The flux ratios we discuss are energy-independent to the extent that the following assumptions are valid: (a)the ratios at production are energy- independent, (b) all oscillations are averaged out, and (c) that all possible decays are complete. In the standard scenario with only oscillations, the final flux ratios are $\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau} = 1 : 1 : 1$. In the cases with decay, we have found rather different possible flux ratios, for

example 4 : 1 : 1 in the normal hierarchy and 0 : 1 : 1 in the inverted hierarchy. These deviations from 1 : 1 : 1 are so extreme that they should be readily measurable.

If we are very fortunate, we may be able to observe a reasonable number of events from several sources (of known distance) and/or over a sufficient range in energy. Then the resulting dependence of the flux ratio (ν_e/ν_μ) on L/E as it evolves from say 4 (or 0) to 1 can be clear evidence of decay and further can pin down the actual lifetime instead of just placing a bound⁴⁹.

To summarize, we suggest that if future measurements of the flavor mix at earth of high energy astrophysical neutrinos find it to be

$$\phi_{\nu_e}/\phi_{\nu_\mu}/\phi_{\nu_\tau} = \alpha/1/1; \quad (8)$$

then

- (i) $\alpha \approx 1$ (the most boring case) confirms our knowledge of the MNSP¹³ matrix and our prejudice about the production mechanism;
- (ii) $\alpha \approx 1/2$ indicates that the source emits pure ν'_μ s and the mixing is conventional;
- (iii) $\alpha \approx 3$ from a unique direction, e.g. the Cygnus region, would be evidence in favor of a pure $\bar{\nu}_e$ production as has been suggested recently⁸;
- (iv) $\alpha > 1$ indicates that neutrinos are decaying with normal hierarchy; and
- (v) $\alpha \ll 1$ would mean that neutrino decays are occurring with inverted hierarchy;
- (vi) Values of α which cover a broader range and deviation of the μ/τ ratio from 1 can yield valuable information about U_{e3} and $\cos\delta$. Deviations of α which are less extreme(between 0.7 and 1.5) can also probe very small pseudo-Dirac δm^2 (smaller than $10^{-12} eV^2$).

Incidentally, in the last three cases, the results have absolutely no dependence on the initial flavor mix, and so are completely free of any dependence on the production model. So either one learns about the production mechanism and the initial flavor mix, as in the first three cases, or one learns only about the neutrino properties, as in the last three cases. To summarize, the measurement of neutrino flavor mix at neutrino telescopes is absolutely essential to uncover new and interesting physics of neutrinos. In any case, it should be evident that the construction of very large neutrino telescopes is a “no lose” proposition.

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References

1. The HESS Collaboration, F. Aharonian et al., astro-ph/0611813.
2. Descriptions of several KM3 detectors can be found at <http://icecube.wisc.edu/> and at <http://www.km3net.org/home.php>
3. The Auger detector is described at <http://www.auger.org/>; for the planned JEM-EUSO detector see <http://jemeuso.riken.jp/> or <http://aquila.lbl.EUSO/>.
4. Examples are ANITA:hep-ph/0503304, and SALSA:astro-ph/0412128.
5. S. Pakvasa, *9th International Symposium on Neutrino Telescopes*, Venice, Italy, 6-9 Mar 2001, Neutrino Telescopes, ed. M. Baldo-Ceolin, Vol. 2, p. 603; hep-ph/0105127.
6. J. G. Learned and S. Pakvasa, *Astropart. Phys.* **3**, 267 (1995), hep-ph/9405296.
7. J. P. Rachen and P. Meszaros, *Phys. Rev.* **D58**, 123005 (1998); astro-ph/9802280; M. Kachelriess, S. Ostapchenko and R. Tomas, *Phys. Rev.* **D77**, 023007 (2008), arXiv:0708.3007.
8. L. Anchordoqui, H. Goldberg, F. Halzen and T. Weiler, *Phys. Lett.* **B593**, 42 (2004), astro-ph/0311002; L. A. Anchordoqui, H. Goldberg, F. Halzen and T. J. Weiler, *Phys. Lett.* **B621**, 18 (2005), hep-ph/041003.
9. S. Razzaque, P. Meozaros and E. Waxman, *Mod. Phys. Lett.* **A20**, 2351 (2005); R. Enberg, M. H. Reno and I. Sarcevic, *Phys. Rev.* **D79**, 053006 (2009); R. Gandhi et al., arXiv: 0905.2483; O. Mena et al. *Phys. Rev.* **D75**, 063003 (2007).
10. K. Greisen, *Phys. Rev. Lett.* **16**, 748 (1966); G. Zatsepin and V. Kuzmin, *JETP Lett.* **4**, 78 (1966).
11. V. S. Berezinsky and G. T. Zatsepin, *Phys. Lett.* **B28**, 423 (1969).
12. R. Engel, D. Seckel and T. Stanev, *Phys. Rev.* **D64**, 093010 (2001), astro-ph/0101216.
13. Z. Maki, M. Nakagawa and S. Sakata, *Prog. Theoret. Phys.* **28**, 870 (1962); B. M. Pontecorvo, *Zh.Eksp.Teor.Fiz.* **53**, 1717(1967); V. N. Gribov and B. M. Pontecorvo, *Phys. Lett.* **B28**, 493 (1969); B. W. Lee, S. Pakvasa, R. Shrock and H. Sugawara, *Phys. Rev. Lett.* **38**, 937 (1977).
14. M. C. Gonzalez-Garcia and M. Meloni, arXiv:0704.1800.
15. P.F. Harrison, D. H. Perkins and W. G. Scott, *Phys. Lett.* **B530**, 267 (2002), hep-ph/0202074.
16. M. Apollonio et al., *Eur.Phys.J.* **C23**31 (2003), hep-ex/0301017.
17. H. Athar, M. Jezabek and O. Yasuda, *Phys. Rev.* **D62**, 103007 (2000); hep-ph/0005104; L. Bento, P. Keranen and J. Maalampi, *Phys. Lett.* **B476**, 205 (2000); hep-ph/9912240.
18. J. F. Beacom, N. Bell, D. Hooper, S. Pakvasa and T. J. Weiler; *Phys. Rev.* **D68**, 093005 (2003), hep-ph/0307025; F. Halzen and D. Hooper, *Rept. Prog. Phys.* **65**, 1025 (2002), astro-ph/0204527.
19. Z-Z. Xing, *Phys. Rev.* **D74**, 013009(2006), hep-ph/065219; W. Rodejohann, *J. Cosmol. Astropart. Phys.* **01**, 029 (2007), hep-ph/0612047; P. D. Serpico and M. Kachelriess, *Phys. Rev. Lett.* **94**, 211102(2005). P. Serpico, *Phys. Rev.* **D73**, 047301 (2006), hep-ph/0511313; S. Choubey, V. Niro and W. Rodejohann, arXiv:0803.0423; K. Blum, Y. Nir and E. Waxman, arXiv:0706.2070; R. L. Awasthi and S. Choubey, *Phys. Rev.* **D76**, 113002 (2007), arXiv:0706.0399; H. Athar, M. Jezabek and O. Yasuda, *Phys. Rev.* **D62**, (2000), hep-ph/0005104.
20. P. Lipari, M. Lusignoli and D. Meloni, *Phys. Rev.* **D75**, 123005 (2007), arXiv:0704.0718; S. Pakvasa, W. Rodejohann and T. J. Weiler, *JHEP* **02**, 05 (2008), arXiv:0711.4517.
21. S. Pakvasa, W. Rodejohann and T. J. Weiler, arXiv: 0802.2008; A. Esmaili and Y. Farzan, arXiv:0905.0259; S. Choubey, W. Rodejohann, arXiv:0909.1219; D. Meloni

and T. Ohlsson, *Phys. Rev.* **D75**, 125017 (2007).

22. D. B. Kaplan, A. E. Nelson and N. Weiner, *Phys. Rev. Lett.* **93**, 091801 (2004), hep-ph/0401099.
23. Super-Kamiokande collaboration, arXiv:0801.0776.
24. J. F. Beacom, N. Bell, D. Hooper, S. Pakvasa and T.J. Weiler, *Phys. Rev. Lett.* **91**, 181301 (2003); hep-ph/0211305.
25. S. Pakvasa, *Physics Potential and Development of Muon Colliders, Mu 99*, ed. D. Cline, San Francisco, *AIP Conf. Proc.* **542** (2000) 99, hep-ph/0004077.
26. G. Gelmini and M. Roncadelli, *Phys. Lett.* **B99**, 411 (1981).
27. Y. Chikasige, R. Mohapatra and R. Peccei, *Phys. Rev. Lett.* **45**, 1926 (1980).
28. A wide variety of models with $\Delta L = 2$ and/or $\Delta L = 0$ for decays of Majorana and Dirac neutrinos have been discussed in the literature: J. Valle, *Phys. Lett.*, **B131**, 87 (1983); G. Gelmini and J. Valle, *ibid.* **B142**, 181 (1983); A. Joshipura and S. Rindani, *Phys. Rev.* **D46**, 3008 (1992); A. Acker, A. Joshipura and S. Pakvasa, *Phys. Lett.* **B285**, 371 (1992); A. Acker, S. Pakvasa and J. Pantaleone, *Phys. Rev.* **D45**, 1 (1992).
29. V. Barger, W-Y. Keung and S. Pakvasa, *Phys. Rev.* **D25**, 907 (1982); A. P. Lessa and O. L. G. Peres, *Phys. Rev.* **D75**, (2007), hep-ph/0701068.
30. K. Hirata et al., *Phys. Rev. Lett.* **58**, 1497 (1988); R.M. Bionta et al., *ibid.* **58**, 1494 (1988).
31. K. Eguchi et al; *Phys. Rev. Lett.* **92**, 071301 (2004), hep-ex/0310047.
32. J. F. Beacom and N. Bell; *Phys. Rev.* **D65**, 113009 (2002), hep-ph/0204111; and references cited therein.
33. V. D. Barger, J. G. Learned, S. Pakvasa and T. J. Weiler, *Phys. Rev. Lett.* **82**, 2640 (1999); hep-ph/9810121; V. Barger, J. G. Learned, P. Lipari, M. Lusignoli, S. Pakvasa and T. J. Weiler, *Phys. Lett.* **B462**, 104 (1999), hep-ph/9907421; Y. Ashie et al., *Phys. Rev. Lett.* **93**, 101801 (2004), hep-ex/0404034.
34. S. Pakvasa, *Lett. Nuov. Cim.* **31**, 497 (1981); Y. Farzan and A. Smirnov, *Phys. Rev.* **D65**, 113001 (2002); hep-ph/0201105.
35. T. J. Weiler, W. A. Simmons, S. Pakvasa and J. G. Learned, hep-ph/9411432.
36. J. F. Beacom, N. Bell, D. Hooper, S. Pakvasa, T.J. Weiler, *Phys. Rev.* **D69**, 017303 (2003), hep-ph/039267.
37. P. Keranen, J. Maalampi and J. T. Peltonieni, *Phys. Lett.* **B461**, 230 (1999), hep-ph/9901403.
38. M. Maltoni and W. Winter, arXiv:0803.2050.
39. K. Enqvist, P. Keranen and J. Maalampi, *Phys. Lett.* **B438**, 295(1998), hep-ph/9806392.
40. G. Domokos and S. Kovesi-Domokos, *Phys. Lett.* **B410**, 57 (1997), hep-ph/9703265.
41. Y. Farzan and A. Yu. Smirnov, arXiv:0803.0495.
42. H. Minakata and A. Yu. Smirnov, *Phys. Rev.* **D54**, 3698 (1996), hep-ph/9601311.
43. V. D. Barger, S. Pakvasa, T. J. Weiler and K. Whisnant, *Phys. Rev. Lett.* **85**, 5055 (2000), hep-ph/0005197.
44. D. Hooper, D. Morgan and E. Winstanley, *Phys. Lett.* **609** 206, (2005), hep-ph/0410094.
45. L. Wolfenstein, *Nucl. Phys.* **B186**, 147 (1981); S. M. Bilenky and B. M. Pontecorvo, *Sov. J. Nucl. Phys.* **38**, 248 (1983); S. T. Petcov, *Phys. Lett.* **B110**, 245 (1982).
46. J. F. Beacom, N. Bell, D. Hooper, J. G. Learned, S. Pakvasa and T. J. Weiler; *Phys. Rev. Lett.*, **92** (2004); hep-ph/0307151; see also P. Keranen, J. Maalampi, M. Myyrylainen and J. Riittinen, *Phys. Lett.* **B574**, 162 (2003), hep-ph/0307041 for similar considerations.
47. M. Kobayashi and C. S. Lim, *Phys. Rev.* **D64**, 013003 (2001), hep-ph/0012266.

48. A. Karle, *Nucl. Phys. Proc. Supp.*, **118** (2003), astro-ph/0209556; A. Goldschmidt, *Nucl. Phys. Proc. Suppl.* **110**, 516 (2002).
49. G. Barenboim and C. Quigg, *Phys. Rev.* **D67**, 073024 (2003), hep-ph/0301220.